4 Open Problems

A number of open problems about self-similar groups and branch groups were raised.

1. Are all contracting self-similar groups amenable? Contracting groups do not have free subgroups (Nekrashevych [6]).

2. Are all automaton groups of polynomial growth amenable? Groups of bounded growth (Bartholdi, Kaimanovich, Nekrasheyvch [1]) and linear growth (Amir, Angel, Virag) are amenable. Groups of polynomial growth do not have free subgroups (Sidki [8]).

3. Is there a residually finite non-amenable group without free subgroups?

4. Is the word problem decidable for finitely generated self-similar groups? How about the uniform problem where the groups are given by functional recursion?

5. Can one decide whether an initial automaton has finite order?

6. Can one decide whether an initial automaton is spherically transitive? (This can be done for n-adic transformations).

7. Can one decide whether an automaton group is infinite?

8. Can one decide whether an automaton group is spherically transitive? (This is decidable for groups of n-adic transformations).

9. Is the word problem for automaton groups PSPACE complete? (It is easy to see that the problem is in PSPACE).

10. Are the products of closed subgroups in the Grigorchuk group closed?

11. Is solvability of equations decidable for the Grigorchuk group?

12. Do contracting groups have decidable conjugacy problem?

13. Do automaton groups have decidable conjugacy problem?

14. Does every hyperbolic group have a faithful self-similar action?

15. Construct self-similar actions of free pro-p groups.

16. What are the kernels of the natural action of fiitely generated algebraically branch groups on rooted trees. In particular, does the center always have finite index in the kernel?

17. Are there finitely generated nonamenable branch groups without free subgroups?

18. Is every maximal subgroup of a finitely generated branch group necessarily of finite index?

19. Do there exist finitely presented branch groups?

20. Is the conjugacy problem decidable in all finitely generated branch groups in which the word problem is decidable?

21. Are all finitely generated hereditarily just infinite groups linear? Do there exist finitely generated, hereditarily just infinite, torsion groups?