where $\mathrm{Neib}(A, R)$ is the set of all vertices that are situated at a distance less than or equal to R from a vertex belonging to A (the R-neighborhood of the set A).

According to the Fölner criterion, a finitely generated group is amenable if and only if its Cayley graph is amenable.

The following theorem is valid (see [1, 34]).

Theorem 6.32. *Let E be a Borel equivalence on the measure space $(\mathcal{X}, \mu)$. If E is hyperfinite and preserves the measure μ, then μ-almost all connected components of any subgraph of bounded valence of equivalence E are amenable.*

A subgraph of equivalence is called a *treeing* if all its connected components are trees.

An example of the equivalence for which there exists a generating treeing is the equivalence E_G, where G is a free group (in particular, a cyclic group) that acts freely. The corresponding treeing is given by the Schreier graph of the group action. Thus, for hyperfinite aperiodic equivalences, there always exists a generating treeing.

On the other hand, not any countable Borel equivalence has a generating treeing. Examples of ergodic group actions for which the orbital equivalence E_G does not have a generating treeing are, by virtue of the results of [2], the actions of groups with the Kazhdan T-property (see also [32]).

Paulin in [51] proved that almost all (in the sense of measure) components of the generating subgraph of a measurable countable equivalence have 0, 1, 2, or an infinite number of ends.

The orbital invariance of L^2-Betti numbers of measurable partitions was proved in [21].

7. LIST OF PROBLEMS

7.1. Growth of automata.

1. Give a classification of the types of growth of noninitial automata.

2. A similar problem for noninitial automata.

3. Calculate the growth asymptotics for concrete automata that have an intermediate growth, for example, the automata depicted in Figs. 8, 9, 14, and 15.

7.2. Groups and semigroups of automata.

1. Does there exist an algorithm that

(a) determines, by a given finite initial automaton, whether or not it is periodic, i.e., whether the equivalence $A_q^{(m+n)} = A_q^{(n)}$ holds for certain $m, n \in \mathbb{N}$?

(b) determines, by a given finite noninitial automaton A, whether the semigroup $S(A)$ (group $G(A)$) is finite, abelian, nilpotent, solvable, free, periodic, or of intermediate growth?

(c) determines, by a given finite noninitial automaton, whether it is an automaton of polynomial, exponential or intermediate growth?

(d) determines, by two automata A and B, whether or not the semigroups $S(A)$ and $S(B)$ are isomorphic? The same question is posed for the groups $G(A)$ and $G(B)$ in the case of invertible automata.

(e) determines whether or not an initial automaton A_q is spherically transitive?

(f) determines whether or not a noninitial automaton A is spherically transitive, i.e., whether the semigroup $S(A)$ is spherically transitive? A similar question for the group $G(A)$ in the case of invertible automata.

(g) determines whether the group $G(A)$ is fractal, branch, weakly branch, or rigid? A group acting on a tree is called *rigid* if the rigid stabilizers of all vertices are finite.

2. Is the conjugacy problem in the groups of finite automata solvable?

3. Does there exist a nonamenable group defined by finite synchronous automata without a free subgroup with two generators? A similar question in the asynchronous case: a candidate for this group is the Thompson group F.

4. Give a classification of groups of finite automata up to the commensurability or to a coarser equivalence defined in [53] (by the commensurability of two groups we mean the situation when these groups have isomorphic subgroups of finite index).

5. Under what conditions on the automorphism group of a homogeneous tree the vertex stabilizer acts by finitely automatic transformations? In particular, is this true for $\mathrm{SL}(2, \mathbb{Z}[1/p])$, where p is a prime number?

Certain other problems of the theory of (synchronous) automatic transformation groups are presented in [46].

7.3. Problems concerning the Schreier graphs $\Gamma(G, S)$.

1. Does there exist an algorithm that

(a) determines, by a given invertible finite automaton A and recursively defined path $w \in \partial T$, whether or not the parabolic subgroup $P_w = \mathrm{St}_G\, w$ is trivial?

(a′) determines if there exists a path w such that $P_w = 1$?

(b) determines whether the Schreier graph $G(A)/P_w$, where w is a recursively defined sequence, has a polynomial growth?

2. Describe all possible types of growth for the graphs $G(A)/P_w$, where A is a finite invertible automaton and P_w is a parabolic subgroup.

3. Does there exist a spherically transitive group of automorphisms of a rooted tree such that the Schreier graphs of orbits, on the boundary, which are typical in the sense of the Baire category (the orbits of generic points) are different from (not locally isomorphic to) the Schreier graphs of orbits which are typical in the sense of measure?

7.4. Problems of the spectral theory of automata.

1. To each automaton, there correspond two spectra: the spectrum of the dynamical system defined by this automaton on the boundary and the spectrum of the Schreier graph. Construct an example of the automaton for which these spectra do not coincide.

2. Give a classification for the topological types of spectra of finite automata.

3. Find new (as compared with those described in [7, 30]) computation methods for the spectra of finite automata.

7.5. Dynamical systems.

1. Give a topological and metric classification of rational homeomorphisms.

2. For a fixed spherically transitive group $G < \mathrm{Aut}\, T$, classify its actions on the boundaries that are induced by the spherically transitive actions of this group on rooted trees.

3. Does there exist an invertible automaton A such that the group $G(A)$ possesses the Kazhdan property? Does there exist an invertible automaton A such that the group $G(A)$ possesses the Kazhdan property and acts ergodically on the boundary of a tree?

4. Construct an invertible automaton A such that the group $G(A)$ is spherically transitive and its action on the boundary defines a nonamenable partition into orbits.

5. Give a classification of orbital equivalences E_G of the action, on the boundary, of the groups of tree automorphisms up to

(a) Borel isomorphisms of the boundary of the tree,

(b) homeomorphisms of the boundary of the tree,

(c) isometries of the boundary of the tree.

6. Give a classification of hyperfinite orbital equivalences E_G of actions on the boundary of the groups of tree automorphisms up to the isometries of the boundary of a tree.

7. Is it true that an arbitrary countable group $G \leq \operatorname{Aut} T$ with the hyperfinite equivalence E_G is conjugate in $\operatorname{Aut} T$ to a subgroup of the group $\mathcal{AWF}(\mathbf{X})$?

8. Does there exist a countable (finitely generated?) amenable subgroup of $\operatorname{Aut} T$ that is not conjugate to a subgroup of the group $\mathcal{AWF}(\mathbf{X})$?

9. Does there exist a cyclic automorphism group of a rooted tree whose orbits on the boundary coincide with the confinality classes?

10. Develop a method for calculating the L^2-Betti numbers for the partitions of a boundary for the actions of the groups $G(A)$, where A is a finite automaton.

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